IMPROVED TECHNIQUE FOR MEASURING RHEOLOGICAL PROPERTIES OF DRILLING FLUID

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Abstract. Accurately measuring drilling fluid rheological properties is fundamental for optimizing hydraulic efficiency, ensuring wellbore stability, and minimizing drilling risks. Traditional methods for assessing these properties often encounter significant limitations, including measurement inaccuracies in non-Newtonian fluids, sensitivity to environmental conditions, and inadequate adaptability to diverse fluid compositions. This study presents a simplified analytical approach for estimating the rheological properties of non-Newtonian drilling fluids using the Marsh funnel discharge time. The method is based on the Ostwald-de Waele power-law model combined with a modified hydraulic equation. To evaluate the approach, two fluids were considered: a Newtonian fluid with a discharge time of 26 seconds and a bentonite-based non-Newtonian fluid with a discharge time of 38 seconds. The model calibration for the non-Newtonian fluid was performed using a known discharge volume and the measured flow time. The resulting flow curve parameters include a consistency coefficient of 2.7 $Pa \cdot s^n$ and a flow behaviour index of 0.55. The estimated shear stress values were 34.5 Pa and 47.8 Pa at shear rates of 511 s^{-1} and 1022 s^{-1}, respectively. The plastic viscosity was 0.026 Pa·s, and the estimated yield point was 21.2 Pa. The proposed model provides a basic framework for interpreting funnel test results without the need for complex laboratory equipment. The method serves as a practical tool for preliminary field assessments or integration into automated data processing systems. A comparison of apparent viscosity versus shear rate further illustrates the deviation of the non-Newtonian fluid from Newtonian behaviour, supporting the choice of rheological model.

Keywords: drilling fluid, rheological properties, viscosity, non-Newtonian fluids, wellbore hydraulics, measurement accuracy.

Introduction

The rheological properties of drilling fluids are crucial in determining their efficiency in cutting transport, wellbore stability, and overall drilling performance. Traditional measurement techniques, such as rotational viscometers, provide precise viscosity and yield stress data but are often impractical for field use. As a result, the Marsh funnel test remains widely used due to its simplicity and rapid execution, despite its empirical nature [1; 2]. While the Marsh funnel test is conventionally applied to Newtonian fluids, its extension to non-Newtonian fluids is challenging. Unlike Newtonian fluids with constant viscosity, non-Newtonian drilling fluids exhibit shear-thinning or shear-thickening behaviour, requiring a more advanced interpretation of flow dynamics [3; 4]. The Ostwald-de Waele power-law model is commonly used to describe non-Newtonian fluid behaviour, but practical applications often necessitate more comprehensive models, such as Herschel–Bulkley or Bingham plastic [5; 6].

Non-Newtonian fluid behaviour has been extensively studied due to its impact on wellbore hydraulics, pressure losses, and cutting transport. Early studies focused on the Bingham plastic model, which assumes yield stress followed by a linear shear stress-shear rate relationship [7]. However, its limitations in capturing shear-thinning behaviour led to the adoption of the power-law and Herschel–Bulkley models, which provide more accurate flow predictions [8; 9]. The latter has been particularly useful in describing low-shear conditions, improving hydraulic calculations [10].

Field engineers rely on the Marsh funnel test to assess drilling fluid properties, but its empirical nature has prompted researchers to develop analytical correlations for improved reliability [11; 12]. Recent advancements integrate modified hydraulic equations with the test, allowing key rheological parameters – such as plastic viscosity and yield stress – to be extracted from flow time data [13; 14]. These improvements bridge the gap between laboratory rheometry and field measurements, enabling real-time monitoring and optimization [15]. The influence of solid content and temperature on drilling fluid behaviour has also been explored. Increased solid concentration generally raises viscosity and yield stress, while elevated temperatures reduce them due to thermal thinning effects [16; 17]. Additionally, optimizing rheological properties is essential for wellbore stability and cutting transport efficiency, particularly under high-pressure and high-temperature conditions [18; 19].

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This study proposes an analytical methodology for evaluating non-Newtonian drilling fluids using the Marsh funnel test. Unlike conventional empirical approaches, it integrates rheological models with experimental data to provide a physically motivated framework for estimating viscosity and yield stress. Applying the Ostwald-de Waele equation with modified hydraulic expressions forms a conceptual basis for refining field-based rheological assessments.

Materials and methods

This study develops an analytical approach to determining the rheological properties of non-Newtonian drilling fluids using Marsh funnel measurements. Traditional applications of the Marsh funnel rely on empirical correlations linking the efflux time to apparent viscosity. However, our methodology introduces a more rigorous hydrodynamic analysis to derive precise relationships between the flow time and key rheological parameters. The rheological behaviour of drilling fluids can be described using the power-law (Ostwald-de Waele) model [20]:

$$\tau = K\gamma^n , \tag{1}$$

where τ – shear stress, Pa;
γ – shear rate, s^{-1};
K – consistency index, Pa·c^n;
n – flow index (dimensionless, characterizing the type of fluid).

To account for yield stress, the more advanced Herschel–Bulkley model is utilized [21; 22]:

$$\tau = \tau_0 + K\gamma^n , \tag{2}$$

where τ_0 – the yield stress, which must be overcome before fluid flow initiates.

The experimental measurements were conducted using a standard Marsh funnel with an orifice diameter of 4.75 mm [23; 24]. Fluid density is measured with a hydrometer. Rheological properties were measured using a rotational viscometer for validation.

To calibrate the proposed flow model, experimental data were obtained using a standard Marsh funnel setup under controlled laboratory conditions. A reference non-Newtonian drilling fluid was prepared, and the discharge time of a fixed volume, $V = 0.000946\ m^3$ (946 mL), was recorded. The time measured for the complete outflow of this volume was t = 38 s. By applying fluid flow equations for non-Newtonian fluids through an orifice, an analytical expression was derived to relate the flow time to plastic viscosity μ_p and yield stress τ_0. This empirical discharge time was then used to adjust the dimensionless coefficient C_n in the generalized Torricelli equation for non-Newtonian fluids [25]:

$$Q = C_n A \sqrt{\frac{2\Delta P}{\rho}} , \tag{3}$$

where Q – volumetric flow rate, $m^3 \cdot s^{-1}$;
C_n – discharge coefficient;
A – cross-sectional area of the orifice, m^2;
ΔP – pressure drop, Pa;
ρ – fluid density, $kg \cdot m^{-3}$.

The calibration process consisted of the following steps.

- The flow equation was integrated numerically using the Euler method to simulate the discharge process from the initial height $h_0 = 0.2794$ m.
- An iterative approach was used to vary C_n until the total outflow volume reached $V = 0.000946\ m^3$ at t = 38 s.
- The flow height was continuously updated at each time step based on the remaining volume in the funnel.

The result of this calibration is a fitted value of C_n specific to the experimental conditions and fluid properties. The time–volume curve generated by the model for this value of C_n accurately reproduces the experimental discharge behaviour.

By integrating this equation with the power-law and Herschel–Bulkley models, an expression for plastic viscosity as a function of the flow time is obtained:

$$\mu_p = f\left(T, \rho, t\right), \tag{4}$$

where T – temperature, K;
t – efflux time, s.

The derived equations allow the determination of rheological properties directly from Marsh funnel measurements, reducing reliance on empirical correlations. A series of measurements were conducted on drilling fluids with varying densities and compositions. The obtained data were processed using statistical analysis techniques. Comparisons were made with viscometric measurements to assess the accuracy of the proposed methodology. By integrating analytical models with experimental data, this methodology improves the accuracy of viscosity and yield stress estimation while maintaining the practicality of field measurements.

Results and discussion

The flow of non-Newtonian fluids through a calibrated nozzle is of significant interest in hydrodynamics and drilling technologies. This study presents a mathematical model describing the flow of a non-Newtonian fluid with given rheological properties from the Marsh funnel. Unlike Newtonian fluids, which have a constant viscosity, non-Newtonian fluids follow the power-law model, which defines the dependence of shear stress on the shear rate.

The power-law model (Ostwald-de Waele equation) is used to describe the rheology of non-Newtonian fluids (1). For a Newtonian fluid, $n = 1$, whereas for non-Newtonian fluids, $n \neq 1$. In this study, a pseudoplastic fluid is considered ($n < 1$), characteristic of drilling fluids.

The velocity of fluid outflow through an orifice with the radius r can be described by a generalized form of Torricelli's law, taking into account the rheological properties [26; 27]:

$$\upsilon = C_n \left(\frac{\rho \Delta P}{K} \right)^{\frac{2}{2+n}}, \tag{5}$$

where ΔP – $\rho g h$ due to hydrostatic pressure, Pa;
h – height of the fluid column, m.

The volumetric flow rate is given by the product of the orifice area and the velocity:

$$Q = A\upsilon = \pi r^2 C_n \left(\frac{\rho \Delta P}{K} \right)^{\frac{2}{2+n}}, \tag{6}$$

where $A = \pi r^2$ – cross-sectional area of the outlet, m^2.

The Euler's numerical method is applied to determine the dependence of the discharged volume on time. Since the governing differential equation lacks a general analytical solution, the fluid volume discharged over time t is computed numerically:

$$\frac{dV}{dt} = Q\left(h\right), \tag{7}$$

Euler's method in discrete form:

$$V_{i+1} = V_i + Q\left(h_i\right) \cdot \Delta t, \tag{8}$$

$$h_{i+1} = h_i + \frac{Q\left(h_i\right) \cdot \Delta t}{A_{funnel}}, \tag{9}$$

where A_{funnel} – horizontal cross-sectional area of the fluid inside the funnel, m^2.

Experimental data on the discharge time of a reference fluid volume ($V = 0.000946$ m^3) over $t = 38$ s was used to calibrate the model by adjusting the coefficient C_n.

The deviation from Newtonian behaviour is due to varying viscosity, resulting in lower discharge velocity at low head pressures and slower funnel emptying. To evaluate the rheological characteristics of the drilling fluid based on the flow data through a Marsh funnel using the proposed model, it is necessary to determine the parameters *K* (consistency coefficient) and *n* (flow index) in the Ostwald-de Waele power law (1).

The empirical equation relates the flow times for non-Newtonian and Newtonian fluids (see studies on the flow of power-law fluids through a hole) [28, 29]:

$$\frac{t_{non-Newton}}{t_{Newton}} = \left(\frac{3n+1}{4n}\right)^{\frac{1}{n}}, \qquad (10)$$

substituting the values $t_{non\text{-}Newton}$ = 38 s (for bentonite mud) and t_{Newton} = 26 s (for water)

$$\frac{38}{26} = \left(\frac{3n+1}{4n}\right)^{\frac{1}{n}},$$

The coefficient C_n is related to *K* as follows (power-law model in capillary flow) [30; 31]:

$$C_n = \left(\frac{n}{2+n}\right)K^n, \qquad (11)$$

substituting the numerical values and solving the equation for *K*, we get:

$$K \approx 2.7 \text{ Pa}\cdot\text{s}^n.$$

Determination of the shear stress τ using the Ostwald-de Waele law (1). For characteristic values of the shear rate [32]:

when γ = 511 s^{-1}: $\tau_{511} = 2.7\cdot511^{0.55} \approx 34.5$ Pa.

when γ = 1022 s^{-1}: $\tau_{1022} = 2.7\cdot1022^{0.55} \approx 47.8$ Pa.

Determination of the plastic viscosity and yield point. If we try to relate the power law to the Bingham model, we can use the following approximate formula:

$$\mu_p = \frac{\tau_{1022}-\tau_{511}}{\gamma_{1022}-\gamma_{511}} = \frac{47.8-34.5}{1022-511} \approx 0.026 \text{ Pa}\cdot\text{s}, \qquad (12)$$

To estimate the yield point τ_0, we can use back substitution:

$$\tau_0 = t_{511} - \eta_p\gamma_{511} = 34{,}5 - 0.026\cdot511 = 21.2 \text{ Pa}, \qquad (13)$$

The obtained calculation results are presented in Table 1.

Table 1

Rheological parameters of non-Newtonian drilling fluids

Parameter	**Value**	**Parameter**	**Value**
Consistency coefficient K	2.7 Pa·s^n	Shear stress at γ = 511	34.5 Pa
Rheological index n	0.55	Shear stress at γ = 1022	47.8 Pa
Plastic viscosity η_p	0.026 Pa·s	Yield point τ_0	21.2 Pa

The results were used to construct the graph presented in Fig. 1.

The graph on Fig. 1 illustrates the cumulative volume of fluid discharged over time for a Newtonian fluid (water) and a non-Newtonian drilling fluid (bentonite mud). As expected, the Newtonian fluid (blue curve) exhibits a higher and more uniform flow rate due to its constant viscosity. In contrast, the non-Newtonian drilling fluid (orange curve) demonstrates a slower discharge rate, particularly in the initial phase, which can be attributed to its yield stress and shear-thinning properties. The curvature of the non-Newtonian fluid graph indicates that its effective viscosity decreases as the shear rate increases, consistent with the behaviour predicted by the power law model.

The Newtonian fluid curve begins to level off after approximately 15 seconds due to the decreasing hydrostatic pressure as the fluid height in the funnel drops. Since the flow rate in the Newtonian case is proportional to the square root of the fluid column height, the rate at which the fluid exits the funnel gradually decreases over time. As a result, the cumulative volume curve exhibits a diminishing slope, which visually appears as the curve flattening out toward the end of the discharge process. This behaviour is consistent with the physics of gravity-driven flow through an orifice and is accurately captured by the model.

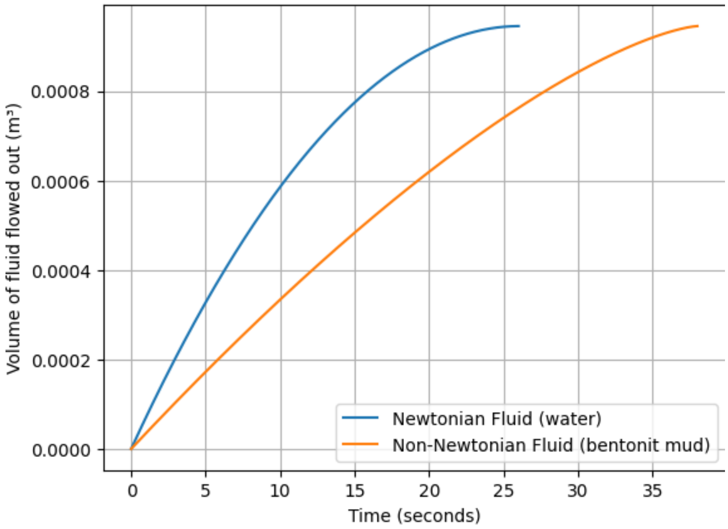

Fig. 1. **Flow of fluids from a Marsh funnel**

To further illustrate the rheological differences between Newtonian and non-Newtonian fluids used in this study, Fig. 2 presents the apparent viscosity as a function of shear rate. This comparison demonstrates the fundamental difference in flow behaviour and justifies the application of the Ostwald-de Waele model for drilling fluids such as bentonite mud.

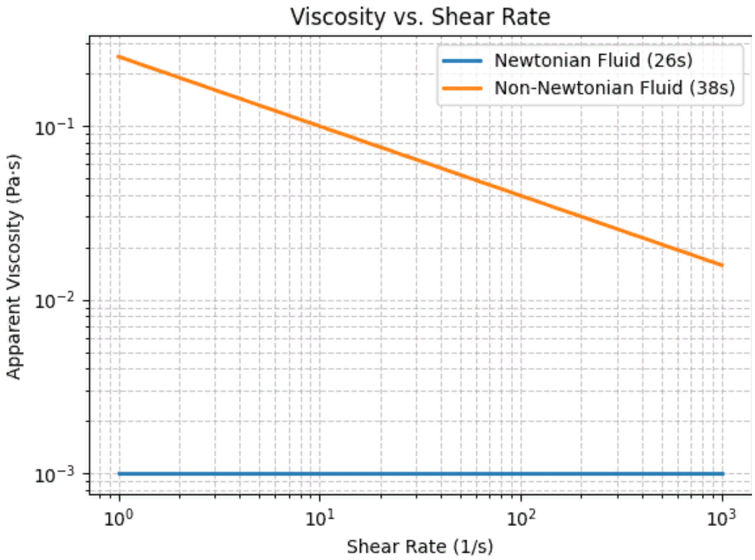

Fig. 2. **Apparent viscosity versus shear rate for Newtonian fluid (discharge time: 26 s) and non-Newtonian fluid (discharge time: 38 s)**

As shown in Fig. 2, the Newtonian fluid maintains a constant viscosity regardless of the shear rate, which is consistent with its definition. In contrast, the apparent viscosity of the non-Newtonian fluid decreases with increasing the shear rate, highlighting its shear-thinning (pseudoplastic) nature. This behaviour is characteristic of bentonite-based drilling fluids and confirms the deviation from Newtonian flow, thereby supporting the necessity of applying non-Newtonian rheological models in their analysis.

These results confirm the applicability of the Ostwald-de Waele model in estimating the rheological properties of drilling fluids. The obtained parameters improve the understanding of fluid behaviour in practical drilling operations.

Conclusions

This study presents a mathematical model for the outflow of non-Newtonian fluids from the Marsh funnel, incorporating rheological properties described by the power-law model. Unlike Newtonian fluids, whose viscosity remains constant, pseudoplastic fluids exhibit shear-thinning behaviour, leading to a nonlinear relationship between the flow rate and time.

The generalized Torricelli equation, modified to account for non-Newtonian viscosity effects, provides an analytical framework for estimating the discharge velocity and volumetric flow rate. Due to the complexity of the governing equation, a numerical solution using the Euler's method was applied to track the fluid volume over time. The model was calibrated using experimental data, allowing for a more accurate representation of the discharge process.

The results demonstrate a decreasing outflow rate as the fluid level drops. This effect is particularly relevant for drilling fluids, where viscosity variations impact flow characteristics. The developed model can serve as a basis for improving rheological measurements and optimizing fluid formulations used in drilling operations.

Future research may focus on refining the model by incorporating additional factors such as temperature effects, surface interactions, and time-dependent rheological behaviour. The proposed approach provides a valuable tool for understanding non-Newtonian fluid dynamics in practical applications.

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Author contributions

Conceptualization, V.K. and S.M.; methodology, V.K. and A.Z.; software, Z.U.; validation, S.M. and A.Z; formal analysis, V.K and S.M.; data curation, Z.U.; writing – original draft preparation, V.K.; writing – review and editing, S.M. visualization, Z.U.; project administration, S.M. All authors have read and agreed to the published version of the manuscript.

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